Thanks for looking at our Pattern Recognition examples

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A table of the types of pattern recognition problems solved in Chapter 2.

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There are three sample problems presented.
They have three levels of difficulty
- Easier
- Moderate
- Harder

Here's the easier one. Scroll or page down for the other two.

Easier Example - Extend the Sequence . . .

Problem: *Write the next two terms of the sequence:*

17, 33, 49, 65, 81, ___, ___

Solution. It is likely that some readers will solve this with an quick "A ha! I get it!". Nice going! But keep on reading. You still need to learn how to go about finding solutions to more difficult sequences — and how to develop formulas for them.

To develop a general solution process, we begin by writing the given sequence with plenty of space above, below, and between the terms — *and* we number the terms beginning with 1:

Term Value:	17	33	49	65	81 . . .
Term Number:	1	2	3	4	5 . . .

The numbers in the sequence (17, 33, 49, etc.) are called the term *values* and given the symbol v(n). The term *numbers* are given the symbol n. In this sequence then, for the third term, n = 3 and v(n) = v(3) = 49.

v(n) ->	17	33	49	65	81 . . .
n ->	1	2	3	4	5 . . . .

With a work sheet set up as shown above, we can begin to look for a pattern to the sequence.

The first pattern to look for in a numerical sequence is *arithmetic* in which the terms differ by a constant amount, either positive or negative. Sometimes the numbers are simple enough that you can check for this possibility mentally. For example: 3, 6, 9, 12 . . . is an arithmetic sequence with a constant difference of +3. Easy. But if the numbers are weird, a pattern may not be readily apparent.

So, above and between the terms, write the operation (start by trying addition or subtraction) required to get from each term to the next. In this case:

		+16		*+16*		*+16*		*+16*	
v(n) ->	17		33		49		65		81 . . .
n ->	1		2		3		4		5 . . . . .

Now it is easy to see that the sequence is arithmetic with a common difference of +16. The sixth and seventh terms are 81 + 16 = 97 and 97 + 16 = 113.

Moderate Examples (a) and (b) - Extend the Sequence, Find the Sum

Problem: (a) *What are the next two rows in this structure, called Pascal's Triangle?*

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
. ...

Solution: Writing out the numbers and a little searching for possibilities usually produces the insight needed to see the pattern, which is about judicious addition.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Each internal number in a row is the sum of the two adjacent numbers in the preceding row.

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(b) *What is the sum of the terms in the 12th row of the Pascal Triangle? (Note: This answer can be found without writing the twelve rows of the triangle, though a calculator may be needed.)*

Solution. Here is a case where it is better to start counting with zero instead of one. That is, count the first row (the lonely 1) as row 0. And so on as shown here:

1 -------- Row 0
1 1 -------- Row 1
1 2 1 -------- Row 2
1 3 3 1 Etc.
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

We could write out all the 12 rows and then add up the terms. But it might be easier and quicker to look for a pattern:

Row	Sum of Terms
0	1
1	2
2	4
3	8
4	16
5	32
6	

This looks to be doubling with each row. The sums are 2^n where n is the row number starting with n = 0. Since 2^{12} = 4096, that's the sum of the terms in the 12th row.

Harder Example - Next Equation?

Problem: Consider this sequence of equations:

$1^2 + 1 = 2$

$2^2 + 2 = 2 + 4$

$3^2 + 3 = 2 + 4 + 6$

$4^2 + 4 = 2 + 4 + 6 + 8$

____

____

What will be the equation in the sixth row? What will be the sum of each side of the equation in the seventh row? (Note: You shouldn't have to write the seventh row to get the sum.)

Solution: In this case, the terms of the sequence are equations, not just numbers or letters. That is, the first *row* is term number n = 1 and the second *row* is n = 2, etc.

n = 1	$1^2 + 1 = 2$
n = 2	$2^2 + 2 = 2 + 4$
n = 3	$3^2 + 3 = 2 + 4 + 6$
n = 4	$4^2 + 4 = 2 + 4 + 6 + 8$
n = 5	________________
n = 6	________________

With this set-up, we see that the left side of the equations are $n^2 + n$.

The right side is different, but pretty obvious when you look at it. The fifth right side will be 2 + 4 + 6 + 8 + 10.

So, the sixth row will be

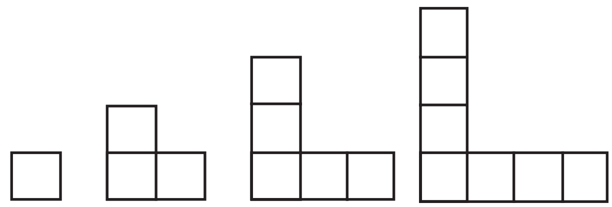
$$6^2 + 6 = 2 + 4 + 6 + 8 + 10 + 12$$

And the sums are found from the usual procedure:

	+4	*+6*	*+8*	*+10*	*+12*	*+14*	*+16*
sums = 2	6	12	20	30	42	56	72
n = 1	**2**	**3**	**4**	**5**	**6**	**7**	**8**

Harder Example - Squares and Rectangles

Problem: How many squares/rectangles will there be in the fifth figure below if it follows the same pattern as the others?



Solution: To see what the pattern is, we have to count the squares and rectangles in each of the first four figures. There are 1, 5, 11, and 19.

Setting this up as a sequence in the usual way, we see the pattern.

	+4	*+ 6*	*+8*	*+10*
sq/rc = 1	5	11	19	
n = 1	**2**	**3**	**4**	**5**

The next number will be 19 + 10 = 29.

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