

Preface

Problem Classes and Solution Processes

Goal

The primary goal of this book is to provide practicing math teachers and math education majors with a practical resource to support the learning and teaching of problem solving skills. However, the book can also be used as a text or reference for short courses and seminars on problem solving. Motivated math students may even learn from it.

Why This Book?

There are many excellent books on solving math problems, and also many sections or chapters on problem solving in algebra books and books for teachers. See the References section (pages R-1 and R-2) that concludes this book. Since such books contain lots of problems and solutions, why *this* book?

There are four features of this book that are intended to make it uniquely useful to teachers and perhaps others:

(1) First, most problem solving books are organized around strategies (e.g., consider a simpler problem, write an equation, work backward, guess and check, etc.) that look for problems to solve. They say ‘Here’s a strategy’ and then ‘Here’s a problem we can solve with this strategy.’

This book is instead organized around classes of problems. That is, this book shows first how to identify types of problems, and then explains in detail the strategies and solution processes used for each class.

(2) A second feature of this book that is uniquely useful is the emphasis on making the solution processes *explicit*. It is not enough to show students how to solve individual problem or even classes of problems; the solution processes *per se* are the focus of the teaching and learning.

(3) The third feature of this book that is uniquely useful is the use of a generalized symbolic analysis process for solving ‘word’ problems, whether they are about ages, mixtures, coins, trains, areas, or other subjects.

(4) There are over 200 solved problems — from easy to relatively difficult — in which the solutions stress the solution process.

Organizing Principle

The core of this book is organized around five classes of problems and their solution processes as shown in the table:

Chapter	Problem Class	Solution Processes
One	Overview of All	
Two	Pattern Recognition	Induction
Three	Counting	Enumeration
Four	‘Word’ Problems	Analysis
Five	Optimization	Search, Graph, Calculus
Six	Recreation	Whatever Works; Try, Test, Revise; Combinations

And there are four appendices on supporting skills and knowledge helpful to math problem solvers:

Appendix	Subject
A	Converting Units
B	‘Per’ Problems
C	Constructing Graphs
D	A Page of Triangles

A section of References concludes the book.

Preface, Continued

It's About the Processes

The problem *classes* in the first table above organize the central chapters, but the emphasis throughout is on the solution *processes*. To the extent possible, we attempt to generalize and make explicit the solution process for each class of problems. The reason is that teachers and students (and students of teaching) should not only be capable problem solvers, but should also understand problem solving processes *explicitly*.

When teaching and learning problem solving, the solution *processes* are key.

Exercises, Puzzles, and Problems

The problems in the book are not exercises. Instructions given for exercises (e.g., “solve for the roots”) make it clear exactly what is to be done. Problems, on the other hand, do not describe what is to be done to find the solution.

Except for one topic (Sudoku), it is not our intent to include puzzles. Puzzles are generally unique and require cleverness or ingenuity (even luck) to solve. In contrast,

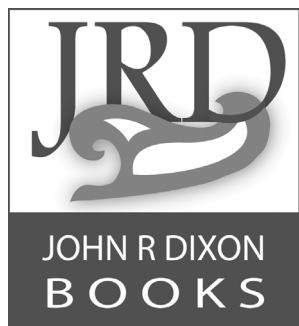
the problems considered here (except for some in the recreational class) are members of identifiable types solved by established methods.

Since puzzles are problems, but not all problems are puzzles, the line between puzzles and problems can occasionally disappear. It's okay — if we solve a few “puzzles”, so be it.

Many Examples — All With Solutions

There are over two hundred examples in the book ranging from easy to moderately difficult. Except in Chapter 1, all problems come with solutions that stress the solution process.

We suggest that readers may want to prepare one or two column-wide cover shields so that problem solutions can be hidden from immediate view while readers consider the solution processes. When the solution is available, it is easy to say, “Sure, I knew that” without really thinking very much about the problem and the solution *process*. And please do remember that the book is not about solutions, but solution *processes*.



Author's Statement to Readers

My primary purpose with this book is to be helpful to teachers. The problem solutions are intended to provide teachers with examples of how solutions, as instances of more general processes, might be presented to students. Thus, many solutions are written *as if* to students. Hopefully, despite the wording, teachers will understand that the solutions are for them — to use as they see fit.

For fifty years, I have been a reflective practitioner of teaching problem solving. I hope that over those years I learned enough, and have been able to put enough of what I learned, into this book so that it helps those still practicing. Your thoughts are always appreciated.

John R. Dixon

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