***Thanks for looking at our Optimization problem examples***

To the Right

Table of the types of optimization problems solved in Chapter 5.

**Types of Organization Problems Solved in Chapter 5**

Optimization With One Variable

- Example 5.1 - Play Yard
- Example 5.2 - Maximize $a^2b^2$
- Example 5.3 - Maximize Profits
- Example 5.4 - Dog Kennels
- Example 5.5 - Cylindrical Tank
- Example 5.6 - Right Triangle
- Example 5.7 - Rectangle
- Example 5.8 - Shortest Time

Optimization With Two Variables

- Example 5.9 - Two Symbolic Variables
- Example 5.10 - Rectangular Tank (Cost)

Linear Programming

- Example 5.10 - Solution at the Corners
- Example 5.11 - Rectangular Tank

Below

Three sample problems are presented.
They have three levels of difficulty
• Easier
• Moderate
•Harder
Scroll or page down for the sample.s





Scroll or Page Down for Easier Example

# Easy Optimization Example

**Problem**: A child's rectangular play yard is to be built next to the house. To make the three sides of the play-pen, twenty-four feet of fencing are available. What should be the dimensions of the sides to make a maximum area?

**Solution:**

**<u>Preparation:</u>** Know the problem thoroughly

- Read the problem statement again, carefully.

*A child's rectangular play yard is to be built next to the house. To make the three sides of the playpen, twenty-four feet of fencing are available. What should be the dimensions of the sides to make a maximum area?*

- Restate the given information clearly

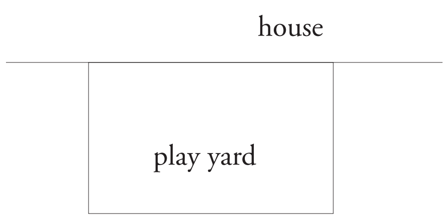
*3 sided play yard - rectangular*
*24 feet of fencing available*
*Make area as large as possible.*

- In words, write what is to be found

*Dimensions of pen for maximum area*

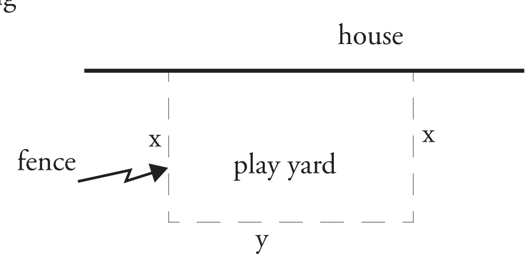
**<u>Translation</u>** into an optimization formulation.

- <u>Drawing (As important as ever!)</u>




- <u>Define Symbols:</u> (Also as important as ever!)

We can add some symbols (and units) to the drawing




- <u>Other Symbols</u>:

A = total area of yard = x y (sq. feet))
L = total length of fence (feet) = 24

- **Now the optimization formulation**:

(a) **Design variables:** x, y
(b) **Criterion Function:** Area = A = x y
(c) **Constraint:** 2 x + y = 24
or y = 24 - 2 x (1)

Note that the constraint (1), in effect, reduces the number of variables from two (x and y) to one: (x).

**<u>Application: Solution by Graphing</u>**

Construct a graph of the criterion function, A, and the (now) single design variable, x. A table of values is:

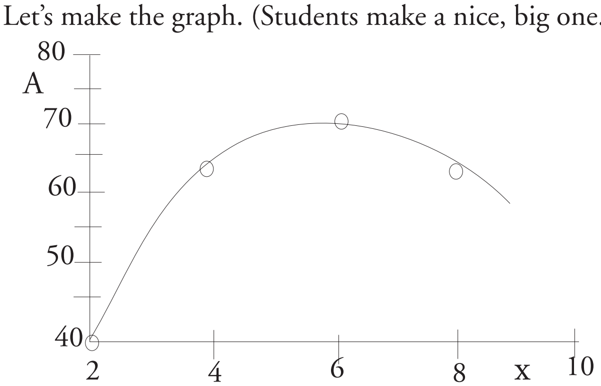
| x (ft) | y = 24 -2x (ft) | $ft^2$ |
|---|---|---|
| 2 | 20 | 40 |
| 4 | 16 | 64 |
| 6 | 12 | 72 |
| 8 | 8 | 64 |

<u>Note:</u> x and y are in feet. A is in $ft^2$.

We can see from just this much that the maximum is between x = 4 and x = 8, so we try some more values between 4 and 8:

| x | y = 24 -2x | A = xy |
|---|---|---|
| 4 | 16 | 64 |
| 5 | 14 | 70 |
| 6 | 12 | 72 |
| 7 | 10 | 70 |

Let's make the graph. (Students make a nice, big one.)




The symmetry around x = 6 leads us to suspect that that the optimum is at x = 6, where A = 72 $ft^2$ . We could verify that assumption with trials at x = 5.9 and 6.1. The maximum is at x = 6 and y = 12 ft.

# Moderate Optimization Example

**<u>Problem:</u>** An open-top cylindrical tank with a volume of ten cubic feet is to be made from a sheet of steel. Find the dimensions of the tank that will require as little material used in the tank as possible.

**<u>Solution:</u>**

**<u>1 - Preparation</u>** Re-reading the problem statement and writing down what is given and what is to be found.

*"An open-top cylindrical tank with a volume of ten cubic feet is to be made from a sheet of steel. Find the dimensions of the tank that will require as little material used in the tank as possible."*

- Rewrite the given info

*A cylindrical tank open at the top.*

*Made from as little sheet material as possible.*

*Volume is 10 $ft^3$ .*

- <u>To Find</u> Dimension of the tank that minimizes the sheet material needed.

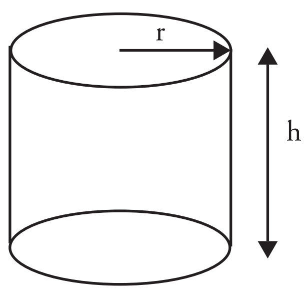
**<u>2 - Translation</u>** to an optimization problem:

"As little material as possible" translates to "as little outside surface area of the tank as possible." Or better: "minimum outside surface area."

- <u>Drawings</u>:

V = Volume = 10 $ft^3$

A = Area
$= \pi r^2 + 2 \pi r h$




<u>- Symbols:</u> Four definitions are in the drawing above.

Volume = V $ft^3$ . Surface Area = A $ft^2$

r = radius of tank (ft) h = height of tank (ft)

**Formulation as an optimization problem**

**Design Variables:** **r, radius (ft)**
**h, height (ft)**

**The Criterion Function in this example is the surface area, A, since when it is minimum, the amount of material used will be a minimum.**

**Criterion Function = A = $\pi r^2 + 2 \pi r h$**

**A Constraint in this case is the required volume.**

**Constraint:** $V = \pi r^2 h = 10 \ ft^3$.

**Or:** $h = 10 / \pi r^2$

Now we have the optimization problem in mathematical form. In words, the math problem is the find the values of r and h that give a tank of minimum surface area with a volume of 10 $ft^3$.

**<u>3 - Application : Solution by Numerical Search</u>.**

We can almost always proceed by systematic numerical search.

In this example, there is really only a single unknown design variable because the constraint on the volume can be used. The table below shows a solution found by trying a value for r, computing h from the volume, and then computing the resulting surface area, A. This table was produced by using a spreadsheet program which is most convenient for such problems, but not really necessary in such a simple one-variable search.

One can start anywhere, but starting with r=1 seems a reasonable place. After the third trial, it is apparent that the minimum A is between r=1 and r=3.

Thus we plan to try r=1.5 and r=2.5. However, the result of r=1.5 reveals that the minimum A will be between r=1 and r=2. We continue with this process as shown.

| Trial | r | $h = 10/\pi r^2$ | $A = \pi r^2 + 20/r$ |
|---|---|---|---|
| 1 | 1 | 3.18 | dec 23.14 |
| 2 | 2 | 0.796 | 22.56 |
| 3 | 3 | 0.35 | inc 34.93 |
| 4 | 1.5 | 1.414 | 20.40 |
| 5 | 1.25 | 2.04 | 20.91 |
| 6 | 1.75 | 1.04 | 21.04 |
| 7 | 1.4 | 1.62 | 20.44 |
| 8 | 1.3 | 1.88 | 20.69 |
| 9 | 1.45 | 1.51 | 20.39 |
| 10 | 1.46 | 1.49 | 20.39 |
| 11 | 1.47 | 1.47 | 20.39 |
| 12 | 1.48 | 1.45 | 20.39 |

The optimum appears to be 20.39 $ft^2$, and it is found at a value of r approximately equal to 1.45 or 1.46 ft. To get more accuracy, we could refine the search.

# Example 5.10 — Search for Two Variables

**Problem:**

Find the values of x and z (both > 0) that maximize

$$U = - x^2 + 10\,x + xz - z^2 + 8z + 2$$

**Solution:**

**Preparation:**

- Re-read it and look at it closely:

*Find the values of x and z (both > 0) that maximize*

$$U = - x^2 + 10\,x + xz - z^2 + 8z + 2$$

Note the two design variables x and z. Note, too, that there are no constraints that can be used to eliminate one of them.

- Rewrite the given info

$$U = - x^2 + 10\,x + xz - z^2 + 8z + 2$$

- To Find: *Values of x and z that maximize U. (x and z are both >0.)*

**Translation** to an optimization problem:

> **Formulation as an optimization problem:**
>
> **Criterion function:**
> $U = - x^2 + 10\,x + xz - z^2 + 8z + 2$
>
> **Design variables:** x, z
>
> **Constraints:** x, z positive

**Application:**

We will solve this using calculus.

(To solve the problem graphically requires three dimensions (x, z, U). There are methods for doing this but they are beyond our interest.)

$$\frac{\partial U}{\partial x} = (-2x + 10 + z) = 0$$

$$\frac{\partial U}{\partial z} = x - 2z + 8 = 0$$

Solving the above two equations for x and z gives:

x = 9.333 and z = 8.667

With these values, U = 83.33.