*Thanks for looking at our*
*Counting Problem Examples*

To the Right

A table of the types of the counting problems solved in Chapter 3.

Below

There are five sample problems presented below. They have levels of difficulty
• Easier
• Moderate
•Harder
Here are two easier ones. Scroll or page down for the others.

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**Easier Example - A Lottery**

**Problem:** *A lottery uses one set of ten ping-pong balls numbered 0 through 9. Three balls are randomly drawn from the set but the balls drawn are not put back into the set. How many possible three-digit numbers can be drawn?*

**Solution:** Apply Principle 1: first draw = 10 possibilities; second draw = 9 possibilities; third draw = 8 possibilities. Thus the total possibilities in this case are 10 *x* 9 *x* 8 = 720.

Counting Principle 1 is extremely general and not difficult to use, but you have to read problems carefully and keep your wits about you to recognize when it applies, when it doesn't, and how to use it without errors. Sometimes the situation calls for the use of Counting Principle 2.



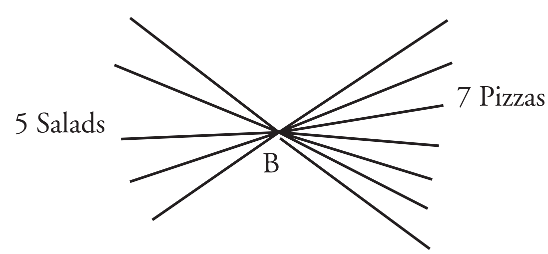
**Types of Counting Problem Examples in Chapter 3**

| Type of Counting Problem Examples | Examples # |
|---|---|
| Counting Principles 1 and 2 | 3.1 - 3.6 |
| Permutations and Combinations | 3.7 - 3.9<br>3.17 - 3.19 |
| Solving by Systematic Enumeration | 3.10 - 3.16 |
| Miscellaneous Other | 3.20 - 3.21 |

**Easier Example - Pizza and Salad**

**Problem:** *Suppose two people go to a restaurant that offers five pizzas and seven salads. Person A wants both a pizza and a salad; therefore by Principle 1, Person A has 7 x 5 = 35 choices for lunch. However, Person B wants either a pizza or a salad. How many choices does Person B have?*

**Solution:** Principle 1 does not apply to person B because the choice is pizza *or* salad, not pizza *and* salad. The situation for B looks like this:




In this case, there are 7 + 5 = 12 choices .

The principle for this case is called Counting Principle 2: *If an event D (e.g., choice of a pizza) can occur in d ways, and another independent event B (e.g., choice of a salad) can occur in b ways, then the total number of ways D or B can occur is:*

*d **plus** b ways*

It is worth remembering: the combined event D *and then* B (sometimes stated as just *D and B*) can occur in d *times* b ways; the event *D or B* can occur in d *plus* b ways.

## Counting Example - Moderate

**Problem:** *How many ways can two cars (red and blue) be arranged in four parking places (A, B, C, D)?*

**Solution:** We're being asked to find the number of permutations; it even says 'arranged'. So let's enumerate the possible *arrangements* (permutations) in a systematic way:

| Arrment | Space A | Space B | Space C | Space D |
|---|---|---|---|---|
| 1 | Red | Blue | | |
| 2 | Red | | Blue | |
| 3 | Red | | | Blue |
| 4 | Blue | Red | | |
| 5 | | Red | Blue | |
| 6 | | Red | | Blue |
| 7 | Blue | | Red | |
| 8 | | Blue | Red | |
| 9 | | | Red | Blue |
| 10 | Blue | | | Red |
| 11 | | Blue | | Red |
| 12 | | | Blue | Red |

Note the system (or pattern) used to insure all cases are considered, but no duplicates. There are other good systems, too. Here's another:

| Arrment | Space A | Space B | Space C | Space D |
|---|---|---|---|---|
| 1 | Red | Blue | | |
| 2 | Blue | Red | | |
| 3 | Blue | | Red | |
| 4 | Blue | | | Red |
| 5 | Red | | Blue | |
| 6 | | Red | Blue | |
| 7 | | Blue | Red | |
| 8 | | Blue | | Red |
| 9 | Red | | | Blue |
| 10 | | Red | | Blue |
| 11 | | | Red | Blue |
| 12 | | | Blue | Red |

Either way, the answer is 12.

## Counting Example - Harder

**Problem:** *A collection of thirteen coins (which may be quarters, nickels, and/or pennies) has a value of 37 cents. How many coins are there and what denominations?*

**Solution:** Anticipating Chapter 4, we could try to do this problem by analysis: Let q = number of quarters, d = number of dimes, etc. Then:

Number of coins, $N = q + d + n + p = 13$

And the sum, $S = 25\,q + 10\,d + 5\,n + p = 37$

The trouble with the analysis method here is that there are four unknowns (q, d, n, and p) but only two equations. We might find a solution by some sort of try, test, and revise process, but it is easier to just do a systematic enumeration — which we know will work.

We enumerate only the ways to make 37 cents, and start with the largest denominations:

| Case | quarters | nickels | pennies | Number of Coins |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 5 |
| 2 | 1 | 1 | 7 | 9 |
| 3 | 1 | 0 | 12 | 13 |
| 4 | 0 | 7 | 2 | 9 |
| 5 | 0 | 6 | 7 | 13 |
| 6 | 0 | 5 | 12 | 17 |
| 7 | 0 | 4 | 17 | 21 |
| 8 | 0 | 3 | 22 | 23 |
| 9 | 0 | 2 | 27 | 29 |
| 10 | 0 | 1 | 32 | 33 |
| 11 | 0 | 0 | 37 | 37 |

The enumeration above shows the total number of ways (11) that 37 cents can be made up with quarters, nickels, and pennies. The ones of these that consist of 13 coins are cases 3 and 5.

(Note: The whole enumeration above didn't have to be done. After the sixth case, it is obvious that the rest will have more than 13 coins.)

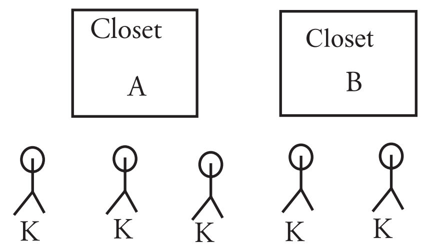


## Counting Example - Harder

**Problem:** *Five children are hiding in two closets. What is the least number of kids in the closet with the most kids?*

**Solution:** Huh? Read that again . . .the *least* number of kids in the closet with the *most* kids?

Now draw a picture to make sure you understand:




The closet with the most kids has to have three, four, or five. The least of these is 3.

Enumerate the possibilities to be sure:

| Case | Closet A | Closet B |
|---|---|---|
| 1 | 5 K's | 0 K's |
| 2 | 4 K's | 1 K's |
| 3 | 3 K's | 2 K's |
| 4 | 2 K's | 3 K's |
| 5 | 1 K's | 4 K's |
| 6 | 0 K's | 5 K's |

Now . . . The *least* number of K's in the closet with the *most* K's . . . In cases 1, 2, and 3, closet A has the most kids, and case 3 has the least of these (3). In cases 4, 5, and 6, closet B has the most kids, and case 4 has the least of these (3). The answer is 3.

**Pigeonhole Principle:** It is possible to live a perfectly useful life without knowing that you know the 'pigeonhole principle.' But here it is anyway:

*Pigeonhole Principle: If m pigeons are placed in n pigeonholes, and m > n, then there must be at least two pigeons in one of the pigeonholes.*

Okay, let's apply it to the kids/closets problem above. The Principle would say: *If m kids are placed in n closets, and m > n, then there must be at least two kids in one of the closets.*

It's true! In every one the cases listed above there is a closet with at least 2 kids.

Now wait a minute! In Case 1, closet B has zero kids; that's less than two. But that's what not the Principle means. It means, considering all the closets, at least one of them must have at least two kids. In case 1, closet A has five kids; that's more than two.