## 5.6 Crossing Over for Fun

There are some famous recreational problems that involve a group of people and/or animals "crossing over" something — a desert, a river, a mountain — with constrained rules, transportation, and/or resources. There is even an extremely good book of fun problems called "Problem Solving Strategies — Crossing the River With Dogs."[1]

Here is an easy such problem; the one following is harder.

### Example 5.6 - Crossing the Ravine

***Problem:*** On a trip to climb a high mountain in Tibet, a group of four amateurs hires three sherpas as guides. Near the top, the group encounters a treacherous ravine that, as explained by the lead sherpa, can be traversed safely only if the following rules are strictly adhered to: Three is the maximum number of people that can cross at one time; two is the minimum. Hikers cannot be left alone without at least one sherpa. What is the most efficient plan for the whole group to get across?

**Solution:**

Looks like a 'word' problem so we'll start off with *preparation* as in our analysis process.

**Preparation** (Get thoroughly familiar with the problem):

\+ Reread the problem - *On a trip to climb a high mountain in Tibet, a group of four amateurs hires three sherpas as guides. Near the top, the group encounters a treacherous ravine that, as explained by the lead sherpa, can be traversed safely only if the following rules are strictly adhered to: Three is the maximum number of people that can cross at one time; two is the minimum. Amateurs cannot be left alone without at least one sherpa. What is the most efficient plan for the whole group to get across?*

\+ Rewrite the given info -

4 amateur hikers and 3 sherpas
Crossing over a ravine
Rules: 3 is maximum that can cross at one time
2 is the minimum
Hikers cannot be left alone without
a sherpa

\+ To find - 'Most efficient' plan for getting the whole group across.

**Translation to Symbolic Representation** (Drawings, symbols, graphs, etc.)

\+ *Draw a diagram to represent the situation as it evolves.*

The importance of creating a diagram that clearly represents the problem and helps with the solution cannot be over-emphasized. It may take several attempts to get a drawing that works well.

Here the ravine is represented as the space between two vertical lines. The direction of crossing is to be from left to right. The beginning state is shown as three shirpas and four hikers on the left

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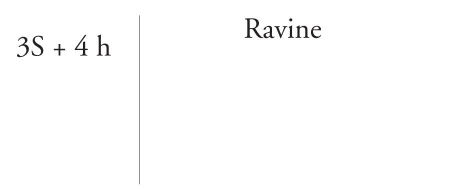


\+ Symbols: Let S stand for a Sherpa
Let h stand for a hiker

**Application** (Apply math and/or science principles and/or rules)

Well, this just hasn't translated into a math or science problem in the usual sense. So, what now?

The list of methods on page 77 doesn't suggest anything except Try, Test, and Revise. Try what?

It seems we have a handle on the beginning, so we'll start there and see what happens.

After just a little 'guess and check', the first crossing by 2 sherpas and 1 hiker seems to be the only one possible by the rules. It can be represented as:

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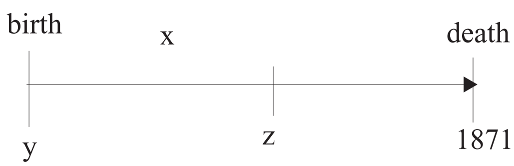
1. *Problem Solving Strategies — Crossing the River With Dogs,* Herr and Johnson, Key Curriculum Press, Berkeley, California, 1994.

### Example 15.8 - Happy Birthday

**Problem:** The logician Augustus De Morgan died in 1871. He once made the statement that he was x years old in the year $x^2$. In what year was he born?[1]

Solution: We might be able to do this by analysis so we do the Preparation. Then . . draw the diagram:, define the symbols, and write the equations:




y = year of birth
x = his age
z = year he made the statement

He said he was x years old in the year $x^2$.

So: $x^2 = z$

And from the diagram: $x = z - y$

We can manipulate these around to get

$$y = x^2 - x \qquad (1)$$

but we can't solve for either y or x unless we know z. There are three unknowns and two equations.

Let's Try Test and Revise. Set up a table to help us. We can try different ages (x) when he made the statement, and find the year of his birth from equation (1). Maybe something will show up.....

Something does show up. (See table below.) Since he died in 1871, his year of birth couldn't be 380, 870, or 1540 corresponding to ages 20, 30, 40. And it couldn't be 2450 either, corresponding to age 50. It has to be more then 40 but less than 50. It turns as the table shows that age 43 is the only one possible: he was born in 1806.

1. For more about this problem, Google 'Augustus DeMorgan.'

| x (Age When He Made Statement) | $x^2$ | $y = x^2 - x$ Year of Birth |
|---|---|---|
| 20 | 400 | 380 |
| 30 | 900 | 870 |
| 40 | 1600 | 1540 |
| 50 | 2500 | 2450 |
| 45 | 2025 | 1980 |
| 42 | 1764 | 1722 |
| 43 | 1849 | 1806 |
| 44 | 1936 | 1892 |

### Example 15.9 - Game of 514

**Problem:** Suppose a poker game called "514" was played with white chips worth five cents and blue chips worth 14 cents. Using those chips, what is the largest bet a player cannot make?[2]

**Solution:** That last sentence again, please: "Using those chips, what is the largest bet a player cannot make?"

After doing the Preparation, this given situation is still not too clear. So let's start playing around just to see what this problem is all about . . .

It is easy to see that a player cannot make bets of 1, 2, 3, or 4 cents, or 6, 7, 8, 9. Or 11, 12, or 13.

Also he can't bet 16, 17, or 18 cents. . . . Okay, enough. This could run on until morning — and beyond.

2. This problem is conceptually similar to the "Brick-layer" problem the author encountered in EDUC 651 taught by Dr. Howard A. Peelle at the University of Massachusetts.