1.3 Arithmetic Sequences

Example 1.1 - Extend the Sequence . . .

Problem: *Write the likely next two terms of the sequence:*

2, 6, 10, 14 ___, ___

Solution. It is possible that many readers will solve this with an quick "Oh, I get it!". The next two terms are 18 and 22. Nice going!

A Process to Try. But let's imagine for a minute that our mind did not quickly induce ("A ha!") that answer. What could we do to help?

Write the sequence in a neat spread out way:

2 6 10 14 __ __

Now, using addition or subtraction, determine how to obtain each term (starting with the second one) from the one before it. Write those numbers (with signs) above and between the terms you have written down:

+4 +4 +4
2 6 10 14 __ __

Well, just about anybody can see the pattern in +4, +4,+4, so the next intervals may be induced to be +4 also.

+4 +4 +4 +4 +4
2 6 10 14 18 22

Note that what we did was find a pattern in the sequence to the way each term is found from the previous term.

1.3.1 Identifying Arithmetic Sequences

A sequence like the one in the example above is called an *arithmetic* sequence. In arithmetic sequences, each term differs from its predecessor by a constant amount which is called the *constant difference* and given the symbol, d. Thus in the previous example, d = +4.

State whether the following are arithmetic sequences and, if so, what are the contant differences?

(a) 6, 13, 20, 27 . . . .

(b) 9, 15, 22, 28 . . . .

(c) 7, 4, 1, -2 . . . .

Answers: (a) is arithmetic with d = 7; the next term is 27+7=34. (b) is not arithmetic. (c) is arithmetic with d = -3. The next term is -2 -3 = -5.

The common difference in an arithmetic sequence can be a fraction or a decimal, but we will deal with such complications later.

Example 1.2 - Extend the Sequence . . .

Problem: *Write the likely next two terms of the sequence:*

103 96 89 82 75 ___, ___

Solution: Using the same technique as in the example above, we write the sequence numbers with plenty of space as before.

103 96 89 82 75 . . . .

Now we try addition and/or substraction to get from one term to the next. And we get:

-7 -7 -7 -7
103 96 89 82 75 . . . .

It is arithmetic with a common difference, d, of -7 so the next two terms are 75 - 7 = 68 and 68 - 7 = 61.

A Do It Yourself Example

Example 1.3 - You Extend the Sequence . . .

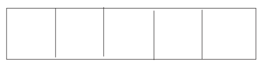
Problem: *Write the likely next two terms of the sequence shown. Also, if it is arithmetice, state the common difference.*

8 17 26 35 44 ___ ___

Solution on page 27.

	+2	*+3*	*+4*
1	3	6	10

The next number will be found by adding 5, so it is 15. Count the squares/rectangles below to be sure:.



A Do It Yourself Example

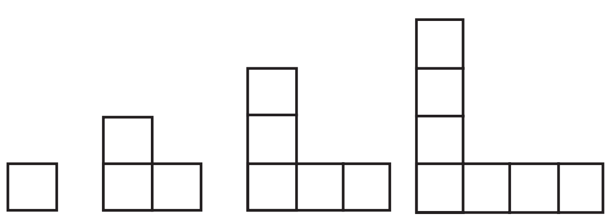
Example 11.13 - You Extend the Sequence . . .

Problem: For the pattern in the previous example, develop a formula for the number of squares/rectangles in the nth figure.

Solution on page 196.

Example 11.14 - Extend the Sequence . . .

Problem: How many squares/rectangles will there be in the fifth figure below if it follows the same pattern as the others? Develop a formula.



Solution: To see what the pattern is, we have to count the squares/rectangles in the first four figures. There are 1, 5, 11, and 19.

Setting this up as usual, we see the pattern.

		+4	*+ 6*	*+8*	*+10*
sq/rc =	1	5	11	19	
n =	**1**	**2**	**3**	**4**	**5**

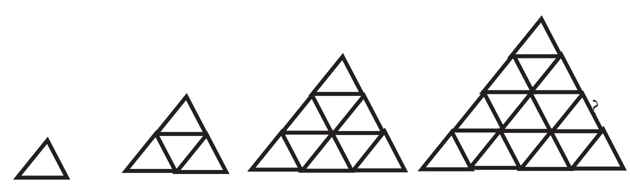
The next number will be 19 + 10 = 29.

v(n) =	1	5	11	19	29
n =	**1**	**2**	**3**	**4**	**5**
n^2 =	1	4	9	16	25
(n-1) =	0	1	2	3	4

The formula is $v(n) = n^2 + (n - 1)$

Example 11.15 - Extend the Sequence . . .

Problem: Write a formula for the number of triangles in the nth figure. Use your formula to find the number in the sixth figure (not shown).



n = 1 2 3 4 ....

Solution:

By counting them (carefully) we find that the number of triangles left to right is 1, 5, 13 and 27. Can't make much of that so we build the next pyramid to get some more data: