

## Example 4.2 — Maximize $a^2b^2$

**Problem:** The difference between two integers,  $a$  and  $b$ , is 6. Integer  $a$  is positive and cannot exceed 6. What are  $a$  and  $b$  when the product of their squares is a maximum?

**Solution:**

**Preparation**

- Read the problem statement again, carefully.

*The difference between two integers,  $a$  and  $b$ , is 6. Integer  $a$  is positive and cannot exceed 6. What are  $a$  and  $b$  when the product of their squares is a maximum?*

- Restate the given information clearly

*Two integers,  $a$  and  $b$   
Integer  $a$  is positive and cannot exceed 6  
Their difference is 6*

- In words, write what is to be found

*Find  $a$  and  $b$  when  $a^2 b^2$  is a maximum*

**Translation:**

- Equations:  $a - b = 6$

- Formulation as an optimization problem:

Design variables:  $a$  and  $b$  (no units)

Criterion function =  $U = a^2 b^2$

Constraints:  $a - b = 6$  so  $b = a - 6$   
 $a$  and  $b$  are integers.  
 $a$  is positive

**Application:** We'll solve two ways: numerical search and calculus. First, this time, by numerical search:

**Method 1) Numerical Search**

Make a convenient table for the search:

$a$	$b$	$a - b$	$a^2 b^2$
1	-5	6	25
2	-4	6	64
3	-3	6	81
4	-2	6	64
5	-1	6	25

That's enough to show the likely optimum is 81 where  $a = 3$  and  $b = -3$ .

**Check:** A graph would do it, but is really obvious.

*A Do It Yourself Example*

## Example 4.A — You Maximize Profits

**Problem:** ABC Marketing has done a study for XYZ Manufacturing and the results show that weekly profits from the sale of a new dustmop will vary with the selling price ( $P$ ) as shown below. At what value should the selling price,  $P$ , be set to maximize profits? Solution on page 75

$$\text{Profits/Week} = -P^3 + 18P^2 + 84P$$

## Solution to ‘Do It Yourself’ Example 9.A.

### Example 9.A You Optimize a Function

**Problem:** Find the optimum value(s) of

$$3t^2 - t^3$$

in the range of  $t$  from  $-5$  to  $+5$ .

**Solution:**

Design variable is:  $t$

The criterion function is:  $U = 3t^2 - t^3$

Constraints are:  $-5 < t < +5$

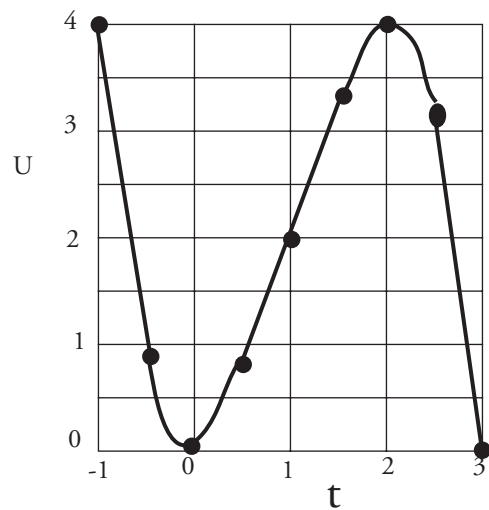
We can make a graph or a table. Since we really need the table values for a graph, let's start with a table.

$t$	$3t^2$	$t^3$	$U = 3t^2 - t^3$
-5	75	-125	200
-4	48	-64	112
-3	27	-27	54
-2	12	-8	20
-1	3	-1	4
0	0	0	0
1	3	1	2
2	12	8	4
3	27	27	0
4	48	64	-16
5	75	125	-50

Well, we see from just this little bit that there is an optimum (minimum) between  $-1$  and  $+1$ . And there is a maximum between  $+1$  and  $+3$ . So let's check some values between  $-1$  and  $+3$ :

$t$	$3t^2$	$t^3$	$U = 3t^2 - t^3$
-1	3	-1	4
-0.5	.75	-.125	.875
0	0	0	0
.5	.75	.125	.625
1	3	1	2
1.5	6.75	3.375	3.375
2	12	8	4
2.5	18.75	15.625	3.125
3	27	27	0

Now we can make a small graph to what this cubic function looks like:



Okay. Minimum at  $t = 0$  and maximum at  $t = 2$ .