2.6 Problems Solved With All Three Enumeration Methods

Example 2.6 - Roll the Dice

Problem: Suppose we roll a pair of honest six-sided dice. How many ways are there to roll a seven?

Solution:

(a) **By List**

Count 7 s	First Die	Second Die	Sum
	1	1	2
	1	2	3
	1	3	4
	1	4	5
	1	5	6
1	1	6	7
	2	1	3
	2	2	4
	2	3	5
	2	4	6
2	2	5	7

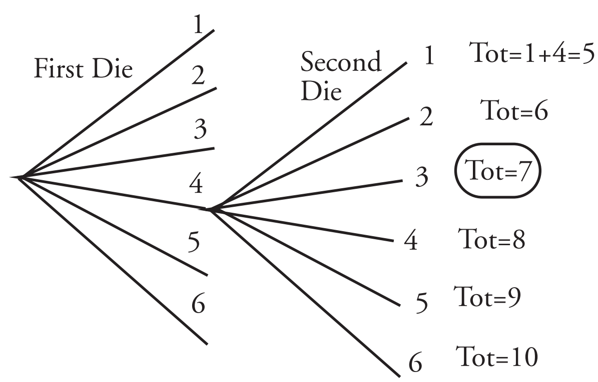
Well, we don't need to finish this list to see the pattern. It would be easy to finish, but long. The answer is six ways to get a seven.

b) By Table

We list the possible numbers for one die along the top, the possible numbers for the second die down the left side, and the sum inside the table. Now it's easy to count the six ways to get a seven.

	1	2	3	4	5	6
1						=7
2					=7	
3				=7		
4			=7			
5		=7				
6	=7					

(c) By Tree Diagram



Again, we have not shown the complete tree diagram, but don't really need to. The answer to how many ways to get a seven is six again.[1]

Example 2.7 - Roll 'em Again

Problem: Suppose we roll a pair of honest six-sided dice. How many ways are there to roll an eight?

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1. Some will recognize that there is a connection here to probability; others may wish to make note of it. There are 36 total outcomes when rolling the two dice, six of which produce a seven. Thus, by fundamental definition of probability, the probability of getting a six on a single roll is 6/36 = 1/6.

Pigeonhole Principle: It is possible to live a perfectly useful life without knowing that you know the 'pigeonhole principle.' But here it is anyway:

Pigeonhole Principle: If m pigeons are placed in n pigeonholes, and m > n, then there must be at least two pigeons in one of the pigeonholes.

Okay, let's apply it to the kids/closets problem above. The Principle would say: *If m kids are placed in n closets, and m > n, then there must be at least two kids in one of the closets.*

It's true! In every one the cases listed above there is a closet with at least 2 kids.

Now wait a minute! In Case 1, closet B has zero kids; that's less than two. But that's what not the Principle means. It means, considering all the closets, at least one of them must have at least two kids. In case 1, closet A has five kids; that's more than two.

Example 12.3 - Changing the Napkins

Problem: *A dinner table has been set with place settings numbered 1 through 10. (Number 1 is the head of the table.) All the places are initially set with blue napkins, but then a waiter changes every setting starting with setting 1 from a blue to a pink napkin. Then a second waiter comes in and changes the color (pink to blue) of settings 2, 4, 6, etc. Then a third waiter changes the color (pink to blue or blue to pink) of every third setting starting with setting number 3. Two more waiters come through and continue the same pattern of changes. Which table settings will then be pink?*

Solution: This is a problem which can be solved by enumerating the colors of the napkins at each setting sequentially as they are, or are not, changed by the waiters. See the table below.

A table something like the one below is necessary for keeping track of the bookkeeping — or in this case, the napkin-changing.

Initially all settings are blue (B). Then the first waiter changes them all to pink (P). Then the second waiter changes the colors at settings 2, 4, 6, 8, and 10. (Shaded blocks in the table below indicate a change was made.) The third waiter changes the positions of settings 3, 6, and 9 only.

And so on. Check it out in the table below.

The final napkin color of the settings is shown in the first table below. The pink ones after five waiters go through will be at settings 1, 4, 6, 7, and 8.

Setting No. >	1	2	3	4	5	6	7	8	9	10
Initially	B	B	B	B	B	B	B	B	B	B
After 1st Waiter	P	P	P	P	P	P	P	P	P	P
After 2nd Waiter	P	B	P	B	P	B	P	B	P	B
After 3rd Waiter	P	B	B	B	P	P	P	B	B	P
After 4th Waiter	P	B	B	P	P	P	P	P	B	P
After 5th Waiter	P	B	B	P	B	P	P	P	B	B